MATRICES AND DETERMINANTS

INTRODUCTORY QUESTION

WHY DO WE STUDY THIS CHAPTER?

WELL you have studied different methods of solving system of equation having two equation in two variables If you have three eqns &three variables then how to solve?

You may be able to solve by reducing it to two eqns in two variables by eliminating one variable from given eqns

But think of a situation with ten eqns in ten variables?

Same method which we employ earlier will be very cumbersome so mathematicians devise ways to overcome this challenge and the branch LINEAR ALGEBRA came into existence which is attributed to SOLVING DIFF TYPE OF SYSTEM OF EQNS

And MATRICES AND DETERMINANTS are the basic concepts

SO LET US START WITH THE CONCEPTS

WHAT IS A MATRIX?

LET US CONSIDER THE RESULT SHEET

NAME	ENG	MATHS	PHY	CHEM	HINDI
X	56	87	86	89	76
Y	65	67	67	70	77
Z	76	98	88	89	89
U	77	89	68	77	79

NAME	X	Y	Z	U
ENG	56	65	76	77
MATHS	87	67	98	89
PHY	86	67	88	68
CHM	89	70	89	77
HIND	76	77	89	79

IF WE CONSIDER THE NUMERICAL PORTION ONLY THEN IT FOMS A MATRIX AND IS WRITTEN AS

$$A=\begin{bmatrix}56 & 87 & 86 & 89 & 76\\ 65 & 67 & 67 & 70 & 77\\ 76 & 98 & 88 & 89 & 89\\ 77 & 89 & 68 & 77 & 79\end{bmatrix}$$

$$B=\begin{bmatrix}56 & 65 & 76 & 77\\ 87 & 67 & 98 & 89\\ 86 & 67 & 88 & 68\\ 89 & 70 & 89 & 77\\ 76 & 77 & 89 & 79\end{bmatrix}$$

SO MATRIX IS AN ORDERED RECTANGULAR ARRAY OF NUMBERS OR FUNCTIONS

- **ALSO YOU KNOW THAT A POINT IN 2-D AND IN 3-D IS REPRESENTED BY AN ORDERED PAIR OR ORDERED TRIPLET LIKE (3,4) OR (1,2,6) THEY CAN ALSO BE REPRESENTED AS [3,4] OR [1,2,6] / $\begin{bmatrix} \mathbf{3} \\ \mathbf{4} \end{bmatrix} \boldsymbol{OR} \begin{bmatrix} \mathbf{1} \\ \mathbf{2} \\ \mathbf{6} \end{bmatrix}$ USING MATRICES**
- **SO MATRIX HAS APPLICATION IN MANY DISCIPLINES**

ORDER

- **NO OF ROWS AND COLS DETERMINES THE ORDER OF THE MATRIX**
- **THE MATRIX A IS OF ORDER 4X5**
- **IF THERE ARE m ROWS AND n COLS THEN ORDER OF MATRIX IS mxn**

Q LET A MATRIX HAS 18 ELTS WHAT ARE THE POSSIBLE ORDERS

POSSIBLE ORDERD WILL BE 2x9/ 3x6 /6x3/9x2/1x18/ 18x1

$$A=\begin{bmatrix} 56 & 87 & 86 & 89 & 76 \\ 65 & 67 & 67 & 70 & 77 \\ 76 & 98 & 88 & 89 & 89 \\ 77 & 89 & 68 & 77 & 79 \end{bmatrix}$$

POSITION

- **IF AN ELT IS LYING IN ith ROW AND jth COL THEN ITS POSITION IS (i,j) th SO GENERAL FORM OF A MATRIX X IS**

j TH COL

$$x = \begin{bmatrix} a_{11} & a_{12} & a_{1j} & a_{1n} \\ a_{21} & a_{22} & a_{2j} & a_{2n} \\ a_{i1} & a_{i2} & a_{ij} & a_{in} \\ a_{m1} & a_{m2} & a_{mj} & a_{mn} \end{bmatrix}$$

i TH ROW

- **THE SUFFIX TELLS THE ROW AND THE COL IN WHICH THE ELT LIE**

- **THE ELT IN i th ROW AND j TH COL IS a_{ij}**

Type equation here.

Q CONSTRUCT A 2x2 MATRIX WHOSE ELTS ARE GIVEN BY a_{ij} = -(i+j)

SOL $a_{11} = -2 \quad a_{12} = -3 \quad a_{21} = -3 \quad a_{22}$ =-4

MATRIX IS $\begin{bmatrix} -2 & -3 \\ -3 & -4 \end{bmatrix}$

TYPES OF MATRICES

- **ROW MATRIX** A MATRIX IN WHICH THERE IS ONLY ONE ROW ITS ORDER WILL BE 1xn

  A= $\begin{bmatrix} 2 & -1 & 5 \end{bmatrix}$ ORDER IS 1x3

  THERE COULD BE ANY NO OF COL.

- **COLUMN MATRIX** A MATRIX IN WHICH THERE IS ONLY ONE COL ITS ORDER WILL BE mx1

  B= $\begin{bmatrix} 2 \\ 5 \end{bmatrix}$ ORDER IS 2x1

  THERE COULD BE ANY NO OF ROWS

- **ZERO MATRIX** A MATRIX IN WHICH AOO ELEMENTS ARE ZERO

  A = $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ B = $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

- **SQUARE MATRIX** A MATRIX IN WHICH NO OF ROWS AND NO OF COL ARE SAME

  $\begin{bmatrix} 2 & 5 & 6 \\ 2 & 0 & -3 \\ 1 & 4 & 8 \end{bmatrix}$ IS A SQ MATRIX OF ORDER 3x3

TYPES CONTINUED

DIAGONAL MATRIX – A SQUARE MATRIX IN WHICH THE ELTS AT MAIN DIAGONAL ARE NON ZERO AND REST OF THE ELTS ARE ZERO IF A=$[a_{ij}]$ IS A DIAG MATRIX THEN $a_{ij} \neq 0$ FOR i= j and a_{ij} =0 FOR i≠j ie $\begin{bmatrix} -1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

- **SCALAR MATRIX** - A SQUARE MATRIX IN WHICH THE ELTS AT MAIN DIAGONAL ARE EQUAL AND REST OF THE ELTS ARE ZERO IF A=$[a_{ij}]$ IS A SCALAR MATRIX THEN a_{ij} =K FOR i= j and a_{ij} =0 FOR i≠j ie $\begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$
- **UNIT MATRIX** - A SQUARE MATRIX IN WHICH THE ELTS AT MAIN DIAGONAL AREUNIT AND REST OF THE ELTS ARE ZERO IF A=$[a_{ij}]$ IS A DIAG MATRIX

  THEN a_{ij} =1 FOR i= j and a_{ij}= 0 FOR i≠ j ie I = $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
- **EQUAL MATRICES** – TWO MATRICE OF SAME ORDER ARE EQUAL IF THE CORRESPONDING ELTS ARE SAME ie IF A=$[a_{ij}]$ B= $[b_{ij}]$ THEN A=B⇒ $a_{i,j}$= $b_{i,j}$ FO R ALL i,j

QUESTIONS

Q1 IF $\begin{bmatrix} 3X & 2Y \\ 2Z & 5W \end{bmatrix} = \begin{bmatrix} X+4 & 6+X+Y \\ -2+Z+W & 2W+3 \end{bmatrix}$ THEN FIND THE VALUE OF X, Y, Z, W

SOL SINCE THE TWO MATRICES ARE EQUAL ⇒ 3X=X+4 ⇒ X=2 2Y= 6+X+Y ⇒ Y =8 5W =2W+3 ⇒ W= 1

2Z = -2+Z+W ⇒ Z = -1

Q2 FIND THE VALUES OF X WHICH MAKES THE TWO MATRICES $\begin{bmatrix} 3X+6 & 5 \\ Y+1 & 2-3X \end{bmatrix}$, $\begin{bmatrix} 0 & Y-2 \\ 8 & -4 \end{bmatrix}$ EQUAL

SOL 3X+ 6= 0 ⟹ X = -6 Y-2 =5 ⇒ $Y = 7$ Y+1 = 8 ⇒ Y = 7 2-3X = -4 ⇒ X = 2

SINCE X CANNOT HAVE TWO VALUES THEREFORE NO VALUE OF X AND Y WILL MAKE THE TWO MATRICES EQUAL

GO THROUGH ALL EXAMPLES

PRACTICE QUESTIONS COMPLETE EX 3.1

HINT Q10 THERE ARE 9 ELTS. IN A 3X3 MATRIX EACH POSITION CAN BE FILLED IN 2 WAYS

NO OF MATRICES = NO OF WAYS = 2X2X--------9 TIMES

OPERATIONS ON MATRICES

- **ADDITION AND SUBTRACTION – TWO MATRICES OF SAME ORDER mxn CAN BE ADDED AND SUBTRACTED BY ADDING THE CORRESPONDING ELTS ie A=$[a_{ij}]$ B= $[b_{ij}]$ THEN A+B= $[c_{ij}]$, $c_{ij} = a_{ij} + b_{ij}$ FOR ALL i,j AND A-B= $[d_{ij}]$, $d_{ij} = a_{ij} - b_{ij}$ FOR ALL i,j**
- **PROPERTIES LET A ,B ,C BE MATRICES OF SAME ORDER**

1 ADDITION IS COMMUTATIVE - A+B = B+A

2 ADDITION IS ASSOCIATIVE - A+(B+C) = (A+B)+C

3 IDENTITY ELEMENT - A+O = A = O+A WHERE O IS ZERO MATRIX OF SAME ORDER

4 EXISTENCE OF NEGATIVE OF A MATRIX – FOR EVERY MATRIX A THERE EXIST A MATRIX –A SUCH THAT

A+(-A) = O = (-A) + A

SCALAR MULTIPLICATION

- **<u>SCALAR MULTIPLICATION</u> – LET k BE ANY SCALAR THEN kA IS A MATRIX OF SAME ORDER IN WHICH EACH ELT IS MULTIPLIED BY k ie**

$$3\begin{bmatrix} 2 & 5 \\ -3 & 2 \end{bmatrix} = \begin{bmatrix} 6 & 15 \\ -9 & 6 \end{bmatrix}$$

- **PROPERTIES LET A AND B BE TWO MATRICES OF SAME ORDER &p q r ARE SCALARS**

1 (pq)A =p(qA)

2 p(A+B) = pA + pB

3 (p+q) A = p A + q A

NOTE – WE CAN TAKE SOME CONSTANT COMMON FROM EACH ELEMENT OF THE MATRIX

$$\begin{bmatrix} 6 & 15 \\ -9 & 6 \end{bmatrix} = 3\begin{bmatrix} 2 & 5 \\ -3 & 2 \end{bmatrix}$$

MULTIPLICATION

FOR MULTIPLYING TWO MATRICES , THE ORDER SHOULD NOT BE SAME

- **MULTIPLICATION** **LET MATRICES A = $[a_{ij}]_{mxn}$ & B = $[b_{ij}]_{n\ xp}$ THEN THE PRODUCT AB IS DEFINED AND IS OF ORDER mxp**
- **The no of columns in A (pre factor) should be same as no of rows of B (post factor)**

HERE THE PRODUCT BA IS NOT DEFINED

symbolically AB = $[c_{ij}]_{mxp}$ WHERE c_{ij} IS THE SUM OF PRODUCT OF CORRESPONDING ELEMENTS of ith ROW OF A AND jth COLUMN OF B i.e.

$$c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + ---- + a_{in}b_{nj}$$

lets take one example

$$AB = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 3 & 2 \end{bmatrix} \begin{bmatrix} 2 & 3 & 7 \\ 4 & 1 & 5 \\ -1 & 3 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 2*2+3*4+4*-1 & 2*3+3*1+4*3 & 2*7+3*5+4*6 \\ 1*2+3*4+2*(-1) & 1*3+3*1+2*3 & 1*7+3*5+2*6 \end{bmatrix}$$

$$= \begin{bmatrix} 12 & 21 & 39 \\ 12 & 12 & 34 \end{bmatrix}$$

Where as the reverse product BA is not defined

PROPERTIES OF MULTIPLICATION

- **MULTIPLICATION IS NOT COMMUTATIVE ,IN GENERAL. - PRODUCTS AB AND BA NEED NOT BE DEFINED BUT ,IF DEFINED, NEED NOT BE EQUAL ALWAYS**
- **AB =** $\begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 2 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 5 & 1 \\ 4 & 3 \end{bmatrix}$

BA = $\begin{bmatrix} 1 & 1 \\ 2 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 3 & 1 \\ 2 & 4 & 0 \\ 2 & 1 & 1 \end{bmatrix} \Rightarrow$ **AB AND BA BOTH ARE DEFINED BUT AB ≠ BA**

$(A+B)^2 \neq A^2 + 2AB + B^2$ **BECAUSE AB≠BA**

$(A+B)^2 = A^2 + AB + BA + B^2$

$(A+B)(A-B) \neq A^2 - B^2$ **BECAUSE AB≠BA**

- **MULTIPLICATION IS ASSOCIATIVE - A(BC) = (AB)C**
- **EXISTENCE OF MULTIPLICATIVE IDENTITY - FOR EVERY MATRIX A THERE EXIST A UNIT MATRIX I SUCH THAT A*I = A = I*A I IS CALLED THE MULTIPLICATIVE IDENTITY**
- **A*O = O , O IS THE ZERO MATRIX**
- **AB = O** $\Rightarrow$ **A = O OR B =O OR** $A \neq O$ **AND** $B \neq O$

QUESTIONS

Q1 FIND AB IF A = $\begin{bmatrix} 0 & 5 \\ 0 & 2 \end{bmatrix}$ B = $\begin{bmatrix} 3 & 2 \\ 0 & 0 \end{bmatrix}$

SOL . AB = $\begin{bmatrix} 0*3+5*0 & 0 \\ 0 & 0 \end{bmatrix}$ = $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ SO A≠ O AND B ≠ O BUT *AB = O*

Q2 FIND THE MATRICES X AND Y IF 2X + 3Y = $\begin{bmatrix} 3 & 8 \\ 8 & 3 \end{bmatrix}$ *X -2Y =* $\begin{bmatrix} -2 & -3 \\ 4 & -2 \end{bmatrix}$

SOL. MULTIPLYING 2ND EQN BY 2 AND SUBTRACTING FROM 1STWE GET 7Y = $\begin{bmatrix} 7 & 14 \\ 0 & 7 \end{bmatrix}$ i.e. Y = $\frac{1}{7}\begin{bmatrix} 7 & 14 \\ 0 & 7 \end{bmatrix}$

NOW PUTTING THE VALUE OF Y IN 2ND EQN ,X =2 $\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} -2 & -3 \\ 4 & -2 \end{bmatrix}$ = $\begin{bmatrix} 2 & 4 \\ 0 & 2 \end{bmatrix} + \begin{bmatrix} -2 & -3 \\ 4 & -2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 4 & 0 \end{bmatrix}$

Q3 IF A = $\begin{bmatrix} 1 & 3 \\ 4 & 2 \end{bmatrix}$ THEN PROVE THAT $A^2 - 3A - 10I = O$

SOL A^2 = $\begin{bmatrix} 1 & 3 \\ 4 & 2 \end{bmatrix}\begin{bmatrix} 1 & 3 \\ 4 & 2 \end{bmatrix}$ = $\begin{bmatrix} 13 & 9 \\ 12 & 16 \end{bmatrix}$ $\Rightarrow L.H.S. = \begin{bmatrix} 13 & 9 \\ 12 & 16 \end{bmatrix} - 3\begin{bmatrix} 1 & 3 \\ 4 & 2 \end{bmatrix} - 10\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

= $\begin{bmatrix} 13-3-10 & 9-9 \\ 12-12 & 16-6-10 \end{bmatrix}$ = $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$= O

PRACTICE QUES. EX 3-2

HINT Q21 PY IS DEFINED SO k=3 AND OF ORDER px3 WY IS DEFINED SO ORDER IS nxk

SINCE PY + WY IS DEFINED SO p = n & k= 3 SOL. (A)

TRANSPOSE OF A MATRIX

- **TRANSPOSE** LET A BE A MATRIX OF ORDER mxn THAN ITS TRANSPOSE IS DENOTED BY A^T

  IT IS OBTAINED BY INTERCHANGING ROWS AND COLUMN

  IF A = $[a_{ij}]_{mxn}$ THEN $A^T = [a_{ji}]_{nxm}$

1. A = $\begin{bmatrix} 2 & 4 & 6 \\ 1 & 3 & 5 \end{bmatrix}$ THEN $A^T = \begin{bmatrix} 2 & 1 \\ 4 & 3 \\ 6 & 5 \end{bmatrix}$ $(A^T)^T = \begin{bmatrix} 2 & 4 & 6 \\ 1 & 3 & 5 \end{bmatrix}$ = A

- **PROPERTIES OF TRANSPOSE**
- $(A^T)^T = A$
- $(A + B)^T = A^T + B^T$ (VERIFY YOURSELF)
- $(AB)^T = B^T A^T$
- AB = $\begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 2 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 5 & 1 \\ 4 & 3 \end{bmatrix}$ $(AB)^T = \begin{bmatrix} 5 & 4 \\ 1 & 3 \end{bmatrix}$
- $B^T A^T = \begin{bmatrix} 1 & 2 & 0 \\ 1 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 2 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 5 & 4 \\ 1 & 3 \end{bmatrix}$ $(AB)^T = B^T A^T$

SPECIAL MATRICES

- **SYMMETRIC MATRIX** - A SQUARE MATRIX A OF ORDER n IS SYMMETRIC IF A^T = A

  SO IN A SYM MATRIX, ELEMENTS EQUIDISTANT FROM MAIN DIAGONAL ARE EQUAl

  I.e. $a_{ij} = a_{ji}$ FOR ALL $1 \leq$ i, j $\leq n$

  LET A = $\begin{bmatrix} 2 & 3 & 5 \\ 3 & 0 & -7 \\ 5 & -7 & 4 \end{bmatrix}$ THEN $A^T = \begin{bmatrix} 2 & 3 & 5 \\ 3 & 0 & -7 \\ 5 & -7 & 4 \end{bmatrix} \Rightarrow$ A^T = A ⇒ A IS SYMMETRIC

- **SKEW SYMMETRIC MATRIX** A SQUARE MATRIX A OF ORDER n IS SYMMETRIC IF A^T = -A
  SO IN A SKEW SYM MATRIX, ELEMENTS EQUIDISTANT FROM MAIN DIAGONAL ARE NEGATIVE OF EACH OTHER i.e. $a_{ij} = -a_{ji,}\ i \neq$ j

  $a_{ii} = -a_{ii} \Longrightarrow 2a_{ii} = 0 \Longrightarrow a_{ii}$ =0 , i=j i.e. DIAGONAL ELTS OF A SKEW SYM MATRIX ARE ZERO

LET A = $\begin{bmatrix} 0 & 2 & -2/3 \\ -2 & 0 & SINX \\ 2/3 & -SINX & 0 \end{bmatrix}$ IS A SKEW SYM MATRIX (VERIFY IT)

IMPORTANT RESULTS

- FOR A SQUAREMATRIX A , (A+A^T) IS SYMMETRIC

$(A + A^T)^T$ =$A^T + \left(A^T\right)^T$ = $A^T + A$ = A+ $A^T \Longrightarrow$ (A+A^T) IS SYMMETRIC

LET A=$\begin{bmatrix} 2 & 6 \\ 3 & 5 \end{bmatrix}$ A+A^T = $\begin{bmatrix} 2 & 6 \\ 3 & 5 \end{bmatrix} + \begin{bmatrix} 2 & 3 \\ 6 & 5 \end{bmatrix}$ =$\begin{bmatrix} 4 & 9 \\ 9 & 10 \end{bmatrix}$ WHICH IS SYMMETRIC

- FOR A SQUAREMATRIX A , (A-A^T) IS SKEW SYMMETRIC

$(A - A^T)^T$ =$A^T - \left(A^T\right)^T$ = $A^T - A$ = -(A- A^T) $\Longrightarrow$ (A-A^T) IS SKEW SYMMETRIC

LET A=$\begin{bmatrix} 2 & 6 \\ 3 & 5 \end{bmatrix}$ $A - A^T$ = $\begin{bmatrix} 2 & 6 \\ 3 & 5 \end{bmatrix} - \begin{bmatrix} 2 & 3 \\ 6 & 5 \end{bmatrix}$ =$\begin{bmatrix} 0 & 3 \\ -3 & 0 \end{bmatrix}$ WHICH IS SKEW SYMMETRIC

- FOR A SQUARE MATRIX A, WE CAN WRITE

2A = (A+A^T) + (A-A^T) $\Longrightarrow$ A = $\frac{1}{2}$(A+A^T) +$\frac{1}{2}$ (A-A^T)

EVERY SQUARE MATRIX CAN BE EXPRESSED AS A SUM OF A SYM ANS A SKEW SYM MATIX

QUESTIONS ON TRANSPOSE

1. IF A AND B ARE SYM MATRICES THEN (AB-BA) IS SKEW SYMMETRIC

   $(\boldsymbol{AB}-\boldsymbol{BA})^{\boldsymbol{T}}$=$(\boldsymbol{AB})^{\boldsymbol{T}}$ -$(\boldsymbol{BA})^{\boldsymbol{T}}$ = $\boldsymbol{B}^{\boldsymbol{T}}\boldsymbol{A}^{\boldsymbol{T}} - \boldsymbol{A}^{\boldsymbol{T}}\boldsymbol{B}^{\boldsymbol{T}}$

   = BA-AB = –(AB-BA)

   ⇒ (AB-BA) IS SKEW SYMMETRIC

2. EXPRESS $\begin{bmatrix} 2 & 4 \\ 1 & 6 \end{bmatrix}$ AS SUM OF A SYM AND A SKEW SYM MATRIX

   LET A = $\begin{bmatrix} 2 & 4 \\ 8 & 6 \end{bmatrix} \Longrightarrow A^T$ =$\begin{bmatrix} 2 & 8 \\ 4 & 6 \end{bmatrix}$

   P = $\frac{1}{2}\left(A+A^T\right)$ = $\frac{1}{2}\left(\begin{bmatrix} 2 & 4 \\ 8 & 6 \end{bmatrix}+\begin{bmatrix} 2 & 8 \\ 4 & 6 \end{bmatrix}\right)$ = $\begin{bmatrix} 2 & 6 \\ 6 & 6 \end{bmatrix}$ WHICH IS SYM (PROVE IT)

   Q = $\frac{1}{2}\left(A-A^T\right)$ = $\frac{1}{2}\left(\begin{bmatrix} 2 & 4 \\ 8 & 6 \end{bmatrix}-\begin{bmatrix} 2 & 8 \\ 4 & 6 \end{bmatrix}\right)$ = $\begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix}$ WHICH IS SKEW SYM (PROVE IT)

   HENCE A = P + Q

3. IF A AND B ARE MATRICES SUCH THAT A^2 =A , B^2 =B &AB = BA = O (ZERO MATRIX)THEN

   PROVE THAT $(A+B)^2 = A+B$ (1 MARK)

   $(\boldsymbol{A}+\boldsymbol{B})^2 = \boldsymbol{A}^2+\boldsymbol{AB}+\boldsymbol{BA}+\boldsymbol{B}^2$ =A + B

3. (HOTS) IF A = $\begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix}$,a,b,c ARE +VE REAL NOS SUCH THAT abc = 1 and A^T A = I THEN FIND THE VALUE OF $a^3 + b^3 + c^3$

$$A^T A = I \Rightarrow \begin{bmatrix} a^2+b^2+c^2 & ab+bc+ac & ab+bc+ac \\ ab+bc+ac & a^2+b^2+c^2 & ab+bc+ac \\ ab+bc+ac & ab+bc+ac & a^2+b^2+c^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$a^2 + b^2 + c^2 = 1$ $\qquad$ $ab + bc + ac = 0$

$(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ac) = 1 \Rightarrow (a + b + c)^2 = 1$

$(a + b + c) = 1$ (a,b,c ARE POSITIVE REAL NOS)

$a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ac) = 1$

$a^3 + b^3 + c^3 = 3abc + 1 = 4$ (abc = 1)

PRACTICE QUESTIONS EX.- 3.3

HOME WORK MISC. EX CH3

DETERMINANTS

- **DETERMINANTS - IT IS A NO. ASSOCIATED WITH A SQUARE MATRIX LET A BE A SQUARE MATRIX OF ORDER n THEN ITS DETERMINANT IS REPRESENTED BY DET A OR $|A|$**

for a 1x1, matrix A =$[a]$ $|A|$ = a

FOR A 2X2 MATRIX , A =$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ $|A|$ = ad-bc

FOR A 3X3 MATRIX , A= $\begin{bmatrix} a & b & c \\ x & y & z \\ p & q & r \end{bmatrix}$ **THEN $|A|$ = a**$\begin{vmatrix} y & z \\ q & r \end{vmatrix} - b\begin{vmatrix} x & z \\ p & r \end{vmatrix} + c\begin{vmatrix} x & y \\ p & q \end{vmatrix}$

= a (y r-q z) – b (x r-p z) + c (x q-y p)

LET A = $\begin{bmatrix} 2 & 1 & 3 \\ 2 & 4 & 1 \\ 3 & 1 & 2 \end{bmatrix}$ **THEN $|A|$ = 2** $\begin{vmatrix} 4 & 1 \\ 1 & 2 \end{vmatrix} - 1\begin{vmatrix} 2 & 1 \\ 3 & 2 \end{vmatrix} + 3\begin{vmatrix} 2 & 4 \\ 3 & 1 \end{vmatrix}$ **=2(8-1)-1(4-3)+3(2-12)**

= 14-1-30 = -17

NOTE – A DETERMINANT CAN BE EVALUATED ALONG ANY COL OR ROW

$|A|$ = -1$\begin{vmatrix} 2 & 1 \\ 3 & 2 \end{vmatrix} + 4\begin{vmatrix} 2 & 3 \\ 3 & 2 \end{vmatrix} - 1\begin{vmatrix} 2 & 2 \\ 3 & 1 \end{vmatrix}$ **(along 2nd col)**

=-1(1)+ 4(-5) -1(-4) = -1-20+4 = -17

PROPERTIES OF DETERMINANTS

- IF ROWS AND COLS OF A MATRIX ARE INTERCHANGED ,THE VALUE OF DET. REMAINS UNCHANGED

  $\begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix}$ = -2 $\begin{vmatrix} 2 & 4 \\ 3 & 5 \end{vmatrix}$ = -2

- IF TWO ROWS (OR COLS)ARE INTERCHANGED ,THE VALUE BECOMES NEGATIVE

  $\begin{vmatrix} 4 & 5 \\ 2 & 3 \end{vmatrix}$ = 2 (Here we interchanged R1 and R2)

- IF ALL THE ELTS OF A ROW (OR COL)ARE ZERO THEN VALUE OF THE DETERMINANT IS ALSO ZERO

  $\begin{vmatrix} 4 & 0 \\ 2 & 0 \end{vmatrix}$ = 0

- IF THE ELTS OF TWO ROWS (OR COLS) ARE SAME 0R PR0P0RTIONAL THEN VALUE OF THE DETERMINANT IS ZERO

  $\begin{vmatrix} 4 & 4 \\ 2 & 2 \end{vmatrix}$ = 0

- IF EACH ELEMENT OF A ROW (OR COL) ARE MULTIPLIED BY A CONSTANT , THE VALUE OF DETERMINANT GETS MULTIPLIED BY THAT CONTANT

  LET $\Delta = \begin{vmatrix} 1 & 2 \\ 4 & 6 \end{vmatrix}$ =-2 AFTER PERFORMING R1$\rightarrow 3R1$ Δ_1 = $= \begin{vmatrix} 3 & 6 \\ 4 & 6 \end{vmatrix}$ =-6 =3* Δ

- IF THE ELTS OF A R0W (OR COL) ARE WRITTEN AS SUM OF TWO OR MORE ELEMENTS THEN THE DETERMINANT CAN ALSO BE WRITTEN AS SUM OF TWO OR MORE DETERMINANTS

- $\begin{vmatrix} a+b & x+y & p+q \\ 1 & 2 & 3 \\ 3 & 4 & 6 \end{vmatrix} = \begin{vmatrix} a & x & p \\ 1 & 2 & 3 \\ 3 & 4 & 6 \end{vmatrix} + \begin{vmatrix} b & y & q \\ 1 & 2 & 3 \\ 3 & 4 & 6 \end{vmatrix}$

- IF , TO THE ELEMENTS OF A ROW(OR COL), MULTIPLE OF CORRESPONDING ELEMENTS OF ANOTHER ROW(OR COL) ARE ADDED THEN THE VALUE OF THE DETERMINANT REMAINS SAME i.e. $R_i \rightarrow R_i + k\ R_j$

$$\begin{vmatrix} a & b & c \\ x & y & z \\ p & q & r \end{vmatrix} = \begin{vmatrix} a+kp & b+kq & c+kr \\ x & y & z \\ p & q & r \end{vmatrix} \qquad (R_1 \rightarrow R_1 + k\ R_3)$$

WE CAN ALSO PERFORM $R_1 \rightarrow R_1 + k\ R_3$ + p R_2 i.e. one row can be changed using the other two .

So using this property we can change the elements of a row or col and the value of the determinant remains same

NOTE IN THE SAME STEPS ,ATMOST TWO ROWS OR COLS CAN BE CHANGED USING THE THIRD ROW

- $|AB| = |A||B|$

SOME MORE SPECIAL MATRICES

- SINGULAR MATRIX A SQ. MATRIX A IS CALLED A SINGULAR MATRIX IF $|A|$ =0
- NON SINGULAR MATRIX A SQ. MATRIX IS A NON SINGULAR MATRIX IF $|A| \neq 0$

EXAMPLES

1 EVALUATE, WITHOUT EXPANDING $\begin{vmatrix} 2 & 3 & 18 \\ 5 & 6 & 39 \\ 7 & 5 & 41 \end{vmatrix}$

$$\begin{vmatrix} 2 & 3 & 18 \\ 5 & 6 & 39 \\ 7 & 5 & 41 \end{vmatrix} = \begin{vmatrix} 2 & 3 & 2*3+3*4 \\ 5 & 6 & 5*3+6*4 \\ 7 & 5 & 7*3+5*4 \end{vmatrix} = \begin{vmatrix} 2 & 3 & 2*3 \\ 5 & 6 & 5*3 \\ 7 & 5 & 7*3 \end{vmatrix} + \begin{vmatrix} 2 & 3 & 3*4 \\ 5 & 6 & 6*4 \\ 7 & 5 & 5*4 \end{vmatrix} = 0 + 0 = 0$$

- 2 PROVE THAT, WITHOUT EXPANDING $\begin{vmatrix} x+y & p+q & a+b \\ y+z & q+r & b+c \\ z+x & r+p & c+a \end{vmatrix} = 2\begin{vmatrix} z & r & c \\ x & p & a \\ y & q & b \end{vmatrix}$

R1$\rightarrow R1 + R2 + R3$

- $\begin{vmatrix} x+y & p+q & a+b \\ y+z & q+r & b+c \\ z+x & r+p & c+a \end{vmatrix} = \begin{vmatrix} 2(x+y+z) & 2(p+q+r) & 2(a+b+c) \\ y+z & q+r & b+c \\ z+x & r+p & c+a \end{vmatrix} =$

$$= 2\begin{vmatrix} (x+y+z) & (p+q+r) & (a+b+c) \\ y+z & q+r & b+c \\ z+x & r+p & c+a \end{vmatrix}$$ (TAKING 2 COMMON FROM R1)

$$=2\begin{vmatrix} x & p & a \\ y+z & q+r & b+c \\ z+x & r+p & c+a \end{vmatrix} \quad (R1 \to R1-R2)$$

$$=2\begin{vmatrix} x & p & a \\ y+z & q+r & b+c \\ z & r & c \end{vmatrix} \quad (R3 \to R3-R1)$$

$$=2\begin{vmatrix} x & p & a \\ y & q & b \\ z & r & c \end{vmatrix} \quad (R2 \to R2-R3) \quad = -2\begin{vmatrix} z & r & c \\ y & q & b \\ x & p & a \end{vmatrix} \quad (R1 \leftrightarrow R3)$$

$$=+2\begin{vmatrix} z & r & c \\ x & p & a \\ y & q & b \end{vmatrix} \quad (R2 \leftrightarrow R3)$$

3 FIND THE VALUE OF x IF $\begin{vmatrix} x & 1 \\ 8 & x \end{vmatrix} = \begin{vmatrix} 2 & 5 \\ -6 & 7 \end{vmatrix}$

$$\begin{vmatrix} x & 1 \\ 8 & x \end{vmatrix} = \begin{vmatrix} 2 & 5 \\ -6 & 7 \end{vmatrix} \Rightarrow \quad x^2+8 = 14+30 \Rightarrow \quad x^2 = 36 \quad \Rightarrow \quad x = \pm 6$$

4 LET A BE A SQUARE MATRIX OF ORDER 2 THEN FIND $|kA|$

$$|kA| = k^2\,|A|$$

LET A= $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ THEN k A = $\begin{bmatrix} ka & kb \\ kc & kd \end{bmatrix} \Rightarrow |kA| = \begin{vmatrix} ka & kb \\ kc & kd \end{vmatrix} = k^2 \begin{vmatrix} a & b \\ c & d \end{vmatrix}$

5 PROVE THAT, WITHOUT EXPANDING $\begin{vmatrix} x+y & y+z & z+x \\ z & x & y \\ 1 & 1 & 1 \end{vmatrix}$ = 0

$$\begin{vmatrix} x+y & y+z & z+x \\ z & x & y \\ 1 & 1 & 1 \end{vmatrix} =$$

$$= \begin{vmatrix} x+y+z & x+y+z & x+y+z \\ z & x & y \\ 1 & 1 & 1 \end{vmatrix} \quad (R_1 \rightarrow R_1 + R_2)$$

= 0 (R1 AND R2 ARE PROPORTIONAL)

6 USING PROPERTIES PROVE THAT

$$\begin{vmatrix} x+4 & 2x & 2x \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix} = (5\mathrm{x}+4)(4-x)^2$$

$$(R1 \rightarrow R1 + R2 + R3)$$

$$\begin{vmatrix} x+4 & 2x & 2x \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix} = \begin{vmatrix} 5x+4 & 5x+4 & 5x+4 \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix}$$

$= (5x+4)\begin{vmatrix} 1 & 1 & 1 \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix} = (5x+4)\begin{vmatrix} 1 & 0 & 0 \\ 2x & 4-x & 0 \\ 2x & 0 & 4-x \end{vmatrix}$ ($C2 \rightarrow C2-C1$, $C3 \rightarrow C3-C1$)

$=(5x-4)((4-x)^2 \begin{vmatrix} 1 & 0 & 0 \\ 2x & 1 & 0 \\ 2x & 1 & 1 \end{vmatrix} = (5x+4))((4-x)^2 (1) == (5x+4))((4-x)^2$

7 IF a, b, c ARE REAL NOS $\begin{vmatrix} a+b & b+c & c+a \\ b+c & c+a & a+b \\ c+a & a+b & b+c \end{vmatrix}$ =0 THEN SHOW THAT EITHER

a = b =c OR (a +b + c) = 0

CONSIDER $\begin{vmatrix} a+b & b+c & c+a \\ b+c & c+a & a+b \\ c+a & a+b & b+c \end{vmatrix}$

$= \begin{vmatrix} 2(a+b+c) & 2(a+b+c) & 2(a+b+c) \\ b+c & c+a & a+b \\ c+a & a+b & b+c \end{vmatrix}$ ($R1 \rightarrow R1+R2+R3$)

$= 2(a+b+c) \begin{vmatrix} 1 & 1 & 1 \\ b+c & c+a & a+b \\ c+a & a+b & b+c \end{vmatrix}$ (2(a+b+c) common from R1)

$= 2(a+b+c) \begin{vmatrix} 1 & 0 & 0 \\ b+c & a-b & a-c \\ c+a & b-c & b-a \end{vmatrix}$ ($C2 \rightarrow C2-C1$ $C3 \rightarrow C3-C1$) $=2(a+b+c)\,[(a-b)(b-a)-(b-c)(a-c)]$

$= -2(a+b+c)\,(a^2 + b^2 + c^2-ab-bc-ac) = -(a+b+c)(2a^2 + 2b^2 + 2c^2-2ab-2bc-2ac)$

$= -(a+b+c)\,[(a-b)^2 + (b-c)^2 + (c-a)^2]$

HENCE WE GET (a +b +c) =0 OR $(a-b)^2 = 0$, $(b-c)^2 = 0$, $(c-a)^2 = 0$

(a +b + c) =0 OR a=b , b=c , c=a i.e. a = b =c OR (a +b + c) = 0

PRACTICE QUES. EX. 4.1 AND 4.2

APPLICATIONS OF DETERMINANTS

- **AREA OF A TRIANGLE** LET A(x_1, y_1),B(x_2, y_2), C(x_3, y_3)BE THE VERTICES OF A TRIANGLE ABC THEN

$$\text{AREA } \Delta ABC = \left| \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) \right|$$

$$= \left| \frac{1}{2}[x_1(y_2 - y_3) - x_2(y_1 - y_3) + x_3(y_1 - y_2)] \right|$$

$$\left| \frac{1}{2} \begin{vmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ 1 & 1 & 1 \end{vmatrix} \right|$$

- **SINGULAR MATRIX** A SQ. MATRIX A IS CALLED A SINGULAR MATRIX IF $|A|$ =0
- **NON SINGULAR MATRIX** A SQ. MATRIX IS A NON SINGULAR MATRIX IF $|A| \neq 0$

1 FIND THE AREA OF THE TRIANGLE WHOSE VERTICES ARE A(1,2) ,B(2,6) AND C(3,8)

AREA ΔABC = $\left| \frac{1}{2}\begin{vmatrix} 1 & 2 & 3 \\ 2 & 6 & 8 \\ 1 & 1 & 1 \end{vmatrix} \right|$ = $\left| \frac{1}{2}[(-2)-(-12)+(-12)] \right|$

= $|-1|$ = 1 sq. units

2 FOR WHAT VALUE OF K THE POINTS (3,K) (2,7) (4,5) ARE COLLINEAR

SINCE THE POINTS ARE COLLINEAR THEREFORE AREA OF THE TRIANGLE IS ZERO

$\begin{vmatrix} 3 & 2 & 4 \\ K & 7 & 5 \\ 1 & 1 & 1 \end{vmatrix}$ =0 ⇒ 6-2(K-5)+4(K-7) = 0 ⇒ 2K-12 = 0 ⇒ K=6

3 FIND THE EQUATION OF THE LINE PASSING THROUGH (2,5) AND (5,-3) USING DETERMINANT

LET P(x,y) BE ANY PT. ON THE LINE JOINING A (2,5) AND B(5,-3)

$AREA\Delta ABP = 0 \Rightarrow \frac{1}{2}\begin{vmatrix} 2 & 5 & x \\ 5 & -3 & y \\ 1 & 1 & 1 \end{vmatrix}$ =0 ⇒2(-3-y) -5(5-y) 8x = 0 ⇒ 8x+3y-31 =0

Q4 IF X, Y, Z ARE NON ZERO AND $\begin{vmatrix} 1+X & 1 & 1 \\ 1 & 1+Y & 1 \\ 1 & 1 & 1+Z \end{vmatrix} = 0$ THEN THE VALUE OF $X^{-1} + Y^{-1} + Z^{-1}$ IS

(A) XYZ (B) $X^{-1}\, Y^{-1}\, Z^{-1}$ (C) -X-Y-Z (D) -1 (D)

Q5 THE SUM OF PRODUCT ELEMENTS OF A COL WITH THE CO- FACTOR OF CORRESPONDING ELEMENTS IS -------------------- (VALUE OF DETERMINANT)

Q6 THE SUM OF PRODUCT ELEMENTS OF A COL WITH THE CO- FACTOR OF ELEMENTS OF ANOTHER COL IS -------------------- (0)

PRACTICE QUES. EX. 4.3

ADJOINT

- **<u>MINOR</u>** - MINOR OF AN ELEMENT OF det A IS THE DETERMINANT OBTAINED BY DELETING THE ROW AND THE COL IN WHICH IT LIE

  MINOR OF a_{ij}= M_{ij}

- **<u>CO-FACTOR</u>** CO-FACTOROF a_{ij} =C_{ij}=$(-1)^{i+j}$ M_{ij}

- **<u>ADJOINT OA A MATRIX</u>** - ADJOINT OF A MATRIX IS THE TRANSPOSE OF THE MATRIX OBTAINED BY REPLACING EACH ELEMENT WITH ITS CO-FACTOR i.e.

  ADJ (A) =$[C_{ij}]^T$

- MINOR OF 3 =$\begin{vmatrix} 1 & 2 \\ 2 & -2 \end{vmatrix} = -6$

- CO FACTOR OF 3 = $(-1)^{1+2}$(-6) = `6

  3 LIES IN 1ST ROW AND 3RD COL

LET A = $[a_{ij}]_{nxn}$

$$|A| = \begin{vmatrix} a_{11} & a_{12} & a_{1j} & a_{1n} \\ a_{21} & a_{22} & a_{2j} & a_{2n} \\ a_{i1} & a_{i2} & a_{ij} & a_{in} \\ a_{m1} & a_{m2} & a_{mj} & a_{mn} \end{vmatrix}$$

- LET A= $\begin{bmatrix} 2 & 3 & 1 \\ 1 & -1 & 2 \\ 2 & 0 & -2 \end{bmatrix}$ detA = $\begin{vmatrix} 2 & 3 & 1 \\ 1 & -1 & 2 \\ 2 & 0 & -2 \end{vmatrix} = 24$

- $\text{ADJ A} = \begin{bmatrix} 2 & 6 & 2 \\ 6 & -6 & 6 \\ 7 & -3 & -5 \end{bmatrix}^T = \begin{bmatrix} 2 & 6 & 7 \\ 6 & -6 & -3 \\ 2 & 6 & -5 \end{bmatrix}$

INVERSE

- **INVERSE OF A MATRIX** _ LET A BE A SQ. MATRIX A IS SAID TO HAVE INVERSE IF THERE EXIST ANOTHER SQ, MATRIX B OF SAME ORDER SUCH THAT AB =BA =I THEN B IS CALLED INVERSE OF A AND IS REPRESENTED AS B= A^{-1}

$$A\,A^{-1} = A^{-1}\,A = I$$

- **IMPORTANT RESULT** - FOR A SQ. MATRIX OF ORDER n ,

$$A\,(\text{Adj } A) = (\text{Adj } A)\,A = |A|\mathbf{I_n} \Rightarrow \quad A\left(\frac{1}{|A|}\text{Adj } A\right) = \left(\frac{1}{|A|}\text{Adj } A\right)A = \mathbf{I_n}$$

COMPARING THE TWO RESULTS, WE CAN SAY

$$A^{-1} = \frac{1}{|A|}\text{AdjA} \quad \text{PROVIDED } |A| \neq 0$$

- HENCE A SQ. MATRIX A IS INVERTIBLE IF AND ONLY IF IT IS NON-SINGULAR

LET $A = \begin{bmatrix} 2 & 3 & 1 \\ 1 & -1 & 2 \\ 2 & 0 & -2 \end{bmatrix}$ THEN Adj A = $\begin{bmatrix} -2 & 6 & 7 \\ 6 & -6 & -3 \\ 2 & 6 & -5 \end{bmatrix} \Rightarrow A^{-1} = \frac{1}{24}\begin{bmatrix} -2 & 6 & 7 \\ 6 & -6 & -3 \\ 2 & 6 & -5 \end{bmatrix}$

$$|A\,AdjA| = \left||A|\mathbf{I}_n\right| \Rightarrow |A||\text{AdjA}| = |A|^n|\mathbf{I}_n|$$

- **RESULT** $|\text{AdjA}| = |A|^{n-1}$ ($|\mathbf{I_n}| = 1$)

$$\left|AA^{-1}\right| = |\mathbf{I}_n| \qquad |A|\,|A^n| = 1$$

- **RESULT** $\left|A^{-1}\right| = \frac{1}{|A|}$

RESULT IF A & B ARE INVERTIBLE MATRICES OF SAME ORDER THEN $(AB)^{-1} = B^{-1}A^{-1}$

Q1 PROVE THAT $(A^{-1})^T = (A^T)^{-1}$

SOL. (A^{-1}) EXISTS $\Rightarrow |A| \neq 0$ $\quad |A| = |A^T| \neq 0$ A^T IS INVERTIBLE

WE KNOW THAT $AA^{-1} = A^{-1}A = I$

$$\Rightarrow (AA^{-1})^T = (A^{-1}A)^T = I^T \Rightarrow (A^{-1})^T A^T = A^T(A^{-1})^T = I$$

$$\Rightarrow (A^T)^{-1} = (A^{-1})^T$$

Q 2 IF A = $\begin{bmatrix} 2 & K & -3 \\ 0 & 2 & 5 \\ 1 & 1 & 3 \end{bmatrix}$ IS INVERTIBLE THEN

(A) K=3 (B) $K \neq 3$ (C) $K \neq -3$ (D) NONE OF THE ABOVE

HINT SINCE A IS INVERTIBLE SO $|A| \neq 0$

Q3 IF A AND B ARE INVERTIBLE MATRICES THEN WHICH STATEMENT IS NOT TRUE

(A) AdjA = $|A|\, A^{-1}$ (B) $|A^{-1}| = |A|^{-1}$ (C) $(A\,B)^{-1} = B^{-1}\,A^{-1}$ (D) $(A+B)^{-1} = B^{-1} + A^{-1}$

SOL. (D) IF TWO MATRICES ARE INVERTIBLE THEN THEIR SUM MAY NOT BE INVERTIBLE

Q4 LET A = $\begin{bmatrix} 2 & 5 \\ 3 & 8 \end{bmatrix}$ $B = \begin{bmatrix} 4 & 3 \\ 6 & 5 \end{bmatrix}$ THEN VERIFY THAT $(AB)^{-1} = B^{-1}A^{-1}$

SOL. $|A| = 1 \neq 0$ $|B| = 2 \neq 0$ $|AB| = |A||B| = 2 \neq 0$ SO A , B , AB ARE INVERTIBLE

AB = $\begin{bmatrix} 38 & 31 \\ 60 & 49 \end{bmatrix}$ Adj (AB) = $\begin{bmatrix} 49 & -31 \\ -60 & 38 \end{bmatrix}$ $(AB)^{-1} = \frac{1}{2}\begin{bmatrix} 49 & -31 \\ -60 & 38 \end{bmatrix}$

$A^{-1} = \begin{bmatrix} 8 & -5 \\ -3 & 2 \end{bmatrix}$ $B^{-1} = \frac{1}{2}\begin{bmatrix} 5 & -3 \\ -6 & 4 \end{bmatrix}$ $B^{-1}A^{-1} = \frac{1}{2}\begin{bmatrix} 5 & -3 \\ -6 & 4 \end{bmatrix}\begin{bmatrix} 8 & -5 \\ -3 & 2 \end{bmatrix}$

$= \frac{1}{2}\begin{bmatrix} 49 & -31 \\ -60 & 38 \end{bmatrix} = (AB)^{-1}$ *HENCE PROVED*

Q5 IF A = $\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ THEN SHOW FIND A^{-1} THAT A^2 -5A+7I= O. HENCE FIND A^{-1}

SOL VERIFY THE EQUATION Det A = $7 \neq 0$ A^{-1} EXISTS

POST MULTIPLYING THE EQN BY A^{-1} , *WE GET* A A A^{-1}-5A A^{-1} + $7I\ A^{-1}$ =O A^{-1}

A I -5 I + 7 A^{-1} = O 7 A^{-1} =5 I- A = $5\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$

$A^{-1} = \frac{1}{7}\begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$ PRACTICE QUESTION EX 4.5

INVERSE USING ROW/COL TRANSFORMATION

- **ROW(/ COL.) TRANSFORMATION ↓**

1 ANY TWO ROWS(C0LS) CAN BE INTERCHANGED i.e. R1↔ R2 or C1 ↔ C2

2 $R_i \rightarrow k\,R_i$ *or* $C_i \rightarrow kC_i$

3 $R_i \rightarrow R_i + k\ R_j$

WE KNOW THAT A = I A (FOR ROW TRANSFORMATION) OR A = A I (FOR COL TRANSFORMATION)

↓ ↓ ↓ ↓

I = B A I = A B

HENCE B = A^{-1}

LET A = $\begin{bmatrix} 7 & 9 \\ 2 & 3 \end{bmatrix}$ NOW A=I A ⇒ $\begin{bmatrix} 7 & 9 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}A \Rightarrow \begin{bmatrix} 1 & 0 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & -3 \\ 0 & 1 \end{bmatrix}A$ $(R1 \rightarrow R1 - 3R2)$

R2→ R2- 2 R1 R2→ $\frac{1}{3}$ R2

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 1 & -3 \\ -2 & 7 \end{bmatrix}A \Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -3 \\ \frac{-2}{3} & \frac{7}{3} \end{bmatrix}A \Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 3 & -9 \\ -2 & 7 \end{bmatrix}A$$

$$\Rightarrow \quad A^{-1} = \frac{1}{3}\begin{bmatrix} 3 & -9 \\ -2 & 7 \end{bmatrix}$$

- **LET US CONSIDER A 3x3 MATRIX**
- $A=\begin{bmatrix}2 & 3 & 1\\ 1 & -1 & 2\\ 2 & 0 & -2\end{bmatrix}$ **WE CAN WRITE** $A = I\,A$ $\begin{bmatrix}2 & 3 & 1\\ 1 & -1 & 2\\ 2 & 0 & -2\end{bmatrix}=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}A$

$R1 \leftrightarrow R2$

$$\begin{bmatrix}1 & -1 & 2\\ 2 & 3 & 1\\ 2 & 0 & -2\end{bmatrix}=\begin{bmatrix}0 & 1 & 0\\ 1 & 0 & 0\\ 0 & 0 & 1\end{bmatrix}A \Rightarrow$$

$R2 \rightarrow R2 - 2R1$ & $R3 \rightarrow R3 - 2R1$

$$\begin{bmatrix}1 & -1 & 2\\ 0 & 5 & -3\\ 0 & 2 & -6\end{bmatrix}=\begin{bmatrix}0 & 1 & 0\\ 1 & -2 & 0\\ 0 & -2 & 1\end{bmatrix}A$$

$R2 \rightarrow R2 - 2R3$

$$\begin{bmatrix}1 & -1 & 2\\ 0 & 1 & 9\\ 0 & 2 & -6\end{bmatrix}=\begin{bmatrix}0 & 1 & 0\\ 1 & 2 & -2\\ 0 & -2 & 1\end{bmatrix}A \Rightarrow$$

$R1 \rightarrow R1 + R2$ & $R3 \rightarrow R3 - 2R2$

$$\begin{bmatrix}1 & 0 & 11\\ 0 & 1 & 9\\ 0 & 0 & 24\end{bmatrix}=\begin{bmatrix}1 & 3 & -2\\ 1 & -2 & -2\\ -2 & -6 & 5\end{bmatrix}A$$

$R3 \rightarrow \frac{-1}{24}R3$

$$\begin{bmatrix}1 & 0 & 11\\ 0 & 1 & 9\\ 0 & 0 & 1\end{bmatrix}=\begin{bmatrix}1 & 3 & -2\\ 1 & -2 & -2\\ \frac{2}{24} & \frac{6}{24} & \frac{-5}{24}\end{bmatrix}A \Rightarrow$$

$R1 \rightarrow R1 - 11R3$ & $R2 \rightarrow R2 - 9R3$

$$\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}=\begin{bmatrix}\frac{2}{24} & \frac{6}{24} & \frac{7}{24}\\ 1 & -2 & \frac{3}{12}\\ \frac{18}{24} & \frac{-30}{24} & \frac{-15}{12}\end{bmatrix}A \Rightarrow \quad A^{-1}=\begin{bmatrix}\frac{2}{24} & \frac{6}{24} & \frac{7}{24}\\ 1 & -2 & \frac{3}{12}\\ \frac{18}{24} & \frac{-30}{24} & \frac{-15}{12}\end{bmatrix}$$

PRACTICE QUESTIONS EX 3.4

SOLVING SYSTEM OF EQUATIONS

- LET US CONSIDER THE FOLLOWING SYSTEM OF EQNS.

  2X+5Y+2Z = -38 , 3X-2Y+4Z =17 , -6X+Y-7Z =-12

  CAN THESE EQNS. BE REPRESENTED IN TERMS OF MATRICES ?

$$\begin{bmatrix} 2X+5Y+2Z \\ 3X-2Y+4Z \\ -6X+Y-7Z \end{bmatrix} = \begin{bmatrix} -38 \\ 17 \\ -12 \end{bmatrix} \quad \text{OR} \quad \begin{bmatrix} 2 & 5 & 2 \\ 3 & -2 & 4 \\ -6 & 1 & -7 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} -38 \\ 17 \\ -12 \end{bmatrix} \quad OR \quad AX = B$$

$\Rightarrow \quad A^{-1} A X = A^{-1} B \quad \Rightarrow I X = A^{-1} B \quad \Rightarrow$ X $= A^{-1}$ B PROVIDED A^{-1} EXISTS

NOW , WHAT IS THE CONDITION FOR A^{-1} TO EXIST $|A| \neq 0$

$|A|$ = -13 $\neq 0$ A^{-1} EXISTS & $A^{-1} = \frac{1}{|A|}(Adj\ A)$

$$A^{-1} = \frac{-1}{13}\begin{bmatrix} 10 & 37 & 24 \\ -3 & -2 & -2 \\ -9 & -32 & -19 \end{bmatrix} \Rightarrow X = \frac{-1}{13}\begin{bmatrix} 10 & 37 & 24 \\ -3 & -2 & -2 \\ -9 & -32 & -19 \end{bmatrix}\begin{bmatrix} -38 \\ 17 \\ -12 \end{bmatrix} = \frac{-1}{13}\begin{bmatrix} -39 \\ 104 \\ 26 \end{bmatrix} = \begin{bmatrix} 3 \\ -8 \\ -2 \end{bmatrix}$$

X = 3 Y= -8 Z = -2 HENCE WE GET A UNIQUE SOL.

WHAT IF $|A|$ =0 ?

CONDITIONS OF SOLVABILITY

LET THE SYSTEM OF EQNS BE REPRESENTED AS A X = B

- SYSTEM OF EQNS. HAVE UNIQUE SOLUTION (CONSISTENT) - $|A| \neq 0$ AND SOL IS GIVEN

BY X $= A^{-1}$ B

$$X = \frac{1}{|A|}(\text{Adj } A)B$$

IF $|A| = 0$ & (Adj A) B = O (ZERO MATRIX) THEN SYSTEM OF EQNS.MAY HAVE INFINITE MANY SOLUTIONS (C0NSISTENT) OR NO SOLUTION (INCONSISTENT)

NOTE WHEN THE SYSTEM HAVE INFINITE MANY SOL THEN EQNS. ARE CALLED DEPENDENT EQNS

IF $|A| = 0$ & (Adj A) B $\neq$ O (ZERO MATRIX) THEN SYSTEM OF EQNS. HAVE NO SOLUTION (INCONSISTENT)

Q

Q EXAMINE THE CONSISTENCY OF FOLLOWING SYSTEM OF EQNS

(1) 2X+3Y = 5   (2) 3X – 4Y = -5   (3) 3X - 5Y = 4

3X-4Y = 1   -6X +8Y = 10   -6X +10Y = 8

(1) SOL. THE SYSTEM CAB BE WRITTEN AS A X = B WHERE A = $\begin{bmatrix} 2 & 3 \\ 3 & -4 \end{bmatrix}$ X = $\begin{bmatrix} x \\ y \end{bmatrix}$ B = $\begin{bmatrix} 5 \\ 1 \end{bmatrix}$

$|A| = \begin{vmatrix} 2 & 3 \\ 3 & -4 \end{vmatrix}$ = -17 $\neq$ 0 $\Rightarrow$ SYSTEM IS CONSISTENT

(2) $|A| = \begin{vmatrix} 3 & -4 \\ -6 & 8 \end{vmatrix}$ = 0 AdjA = $\begin{bmatrix} 8 & 4 \\ 6 & 3 \end{bmatrix}$ (AdjA)B = $\begin{bmatrix} 8 & 4 \\ 6 & 3 \end{bmatrix} \begin{bmatrix} -5 \\ 10 \end{bmatrix}$ = $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\Rightarrow$ SYSTEM IS CONSISTENT WITH INFINITE SOLS.

(3)) $|A| = \begin{vmatrix} 3 & -5 \\ -6 & 10 \end{vmatrix}$ = 0 AdjA = $\begin{bmatrix} 10 & 5 \\ 6 & 3 \end{bmatrix}$ (AdjA)B = $\begin{bmatrix} 10 & 5 \\ 6 & 3 \end{bmatrix} \begin{bmatrix} 4 \\ 8 \end{bmatrix}$ = $\begin{bmatrix} 80 \\ 48 \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\Rightarrow$ SYSTEM IS INCONSISTENT

PRACTICE QUESTION EX 4.6

HOME WORK MISC. EX CH 4